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February 1982

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C.T. Chang

ERRATA:

Page 7, Eq. (7) : (L(E) , should read: L(E)

Page 10, Eq. (25): (E) , " " : (E)

Page 16, 1st line, 2nd paragraph: N_a, should read: $\dot{N}_a$

Page 20, last line, " " : 10^o, " " : 20^o

Page 28, section 5, 2nd line: Trunbull, " " : Turnbull

Page 29, delete the last line

Page 30, delete the first line and the first word in the second line

RISØ-R-460

ON THE BOUNDARY CONDITIONS AND VALIDITY OF THE NEUTRAL SHIELDING MODEL OF A REFUELLING PELLET

C.T. Chang

Abstract. By comparing the ablation time of a hydrogen pellet in a tokamak discharge with the time required for the sublimation process, the vaporization of the pellet is shown to be a dynamic phase transition - i.e. the transport of heat is due to the propagation of an evaporation front. Based on this finding, an alternative boundary condition, consistent with the energy conservation law, is formulated.

Computational results ultilizing the new boundary condition indicate that the ablatant near the pellet surface is hotter and less dense compared with the results which make use of the previous condition of the vanishing flux. The discrepancy between the two solutions becomes less significant once the ablatant reaches the sonic radius. The scaling law of the pellet ablation rate is unaffected by this change of boundary condition. The present analysis shows that the validity of the neutral shielding model is based mainly on the existence of a thin envelope around the pellet where strong energy absorption occurs and is insensitive to the actual vaporization process occuring at the pellet surface.

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CONTENTS

1. INTRODUCTION

The injection of a pellet of hydrogen isotopes was considered in the very beginning of the controlled thermonuclear fusion research program by Spitzer et al.[1] as a possible means of refuelling future fusion reactors. Pellet ablation experiments[3-7] are currently pursued in several laboratories under various tokamak discharge conditions. Among many shielding models proposed[2] for the ablation process, the neutral shielding model[8-12] seems to agree well, at least qualitatively, with present tokamak discharge conditions.

According to this model, after the pellet has been subjected to the direct impact of energetic plasma electrons, a protective cloud of neutral particles immediately forms around the pellet. Through the inelastic and elastic collision process of slowing down and the scattering of the incoming plasma electrons by the neutral particles present in the cloud, the pellet lifetime is significantly prolonged. In other words, the ablated cloud serves as a stopping medium for the incoming plasma electrons. However, the cloud differs from the usual stopping medium of a solid target because of the regulation of the density of the stopping medium by the hydrodynamics of the expanding cloud. The expansion of the cloud depends, in turn, on the way the energy of the incoming electrons is deposited at the pellet surface. For this reason, one might expect that the precise vaporization process occurring at the pellet surface would affect the expansion process of the ablatant, which might then influence the energy absorption process of the cloud. In order that the shielding be effective, the electron energy flux, q_p, received at the pellet surface is expected to be small. As the evaporate is in close contact with the frozen hydrogen pellet, the temperature T_v, of the evaporate at the pellet surface is expected to be low. In comparison with their respective values of q_* and T_* at the sonic radius of expansion, Parks and Turnbull[11] there-

therefore assumed $\hat{q} = q_p/q_* = 0$ and $\hat{T} = T_v/T_* = 0$ as the inner boundary condition for the expansion process. From their formulation of the shielding model, it can be seen that this boundary condition implicitly assumes the vaporization of the the pellet to be a sublimation process.

Extensive computations based on the shielding model of Parks and Turnbull showed that for present and future tokamak discharges the pellet ablates in a time much shorter than that spent for the sublimation process. Accordingly, it would be interesting to see what could be the alternative vaporization process under most tokamak discharge conditions, and to enquire what would be the corresponding boundary condition at the pellet surface in place of the vanishing energy flux condition, $\hat{q} = 0$. As a practical consideration of such an investigation, one is naturally prompted to ask to what extent the ablated flow and, in particular, the scaling law of the pellet ablation rate would be affected by such a change of boundary condition at the pellet surface.

2. THE SHIELDING MODEL OF PARKS AND TURNBULL

2.1. Governing equations and boundary conditions

As a basis for further discussion, in the following we will present a brief review of the shielding model of Parks and Turnbull[11].

The main assumptions of their model are the following:

(i) The Maxwellian distribution of plasma electrons is replaced by a mono-energetic beam of the same number density and power flux.

(ii) The ablatant is taken as an ideal gas with a constant ratio of specific heat at constant pressure to constant volume, γ.

(iii) The expansion is spherically symmetric, and quasi-stationary.

(iv) A constant fraction of the incident electron energy flux, independent of its radial position, is spent in heating and expanding the ablatant.

With these assumptions, the expansion of the ablatant can be described by the follwing system of governing equations:

$$p = \rho\frac{kT}{m} \tag{1}$$

$$\rho v r^2 = \frac{G}{4\pi} \tag{2}$$

$$\rho v \frac{dv}{dr} = -\frac{dp}{dr} \tag{3}$$

$$\frac{G}{4\pi r^2}\frac{d}{dr}\{\frac{\gamma}{\gamma-1}\frac{kT}{m} + \frac{v^2}{2}\} = \frac{dq}{dr} \tag{4}$$

$$\frac{dq}{dr} = \frac{\rho}{m} q\Lambda E) \tag{5}$$

$$\frac{dE}{dr} = 2\frac{\rho}{m} L(E) \tag{6}$$

$$\Lambda(E) = 2\frac{(L(E)}{E} + \sigma(E) \tag{7}$$

In the above system of equations, m is the mass of hydrogen molecules, G the mass ablation rate, and L(E) the loss function.

The effective cross section $\Lambda(E)$ for the energy attenuation contains two terms: the inelastic collision term is represented by the stopping cross section, $2L(E)/E$, and the elastic collision term is represented by the scattering cross section, $\sigma(E)$. Thus,

$$\sigma(E)[\mathrm{cm}^2] = 8.8\times10^{13}E^{-1.71} - 1.62\times10^{-12}E^{-1.932} \qquad \text{for } E > 100 \text{ eV}$$

$$= 1.13\times10^{-14}\ E^{-1} \qquad \text{for } E < 100 \text{ eV} \tag{8}$$

$$L(E)[\mathrm{eV - cm}^2] = \{2.35\times10^{14} + 4\times10^{11}E + 2\times10^{17}E^{-2}\}^{-1}. \tag{9}$$

For simplicity, we have replaced the original expression of $L(E)$ in ref. 11 by the smooth function used in ref. 12.

Denoting the prameters at the sonic radius of expansion by an asterisk, the above system of equations can be expressed in terms of their corresponding dimensionless variables, e.g. $r' = r/r_*$, $p' = p/p_*$, $\rho' = \rho/\rho_*$, $\Lambda' = \Lambda(E)/\Lambda(E_*)$, etc.

By eliminating p' and ρ' we can replace the momentum and energy equations by two coupled equations describing v' and T'. They express, respectively, the change of the thermal and kinetic energies of the ablated flow; thus,

$$\frac{dT'}{dr'} = 2\frac{\Lambda'q'}{v'} - (\gamma-1)v'\frac{dv'}{dr'} \ . \tag{10}$$

$$\frac{d}{dr}\left(\frac{v'}{2}\right)^2 = \frac{T'v'^2}{(T'-v'^2)}\left[\frac{A_*\lambda_*}{2}\,\frac{\Lambda'q'}{T'v'} - \frac{1}{r'}\right] \ , \tag{11}$$

where

$$\lambda_* = \rho_*\Lambda_*r_*/m \ , \tag{12}$$

$$\Lambda_* = 4\pi r_*^2\left(\frac{m}{G}\right)\left(\frac{\gamma}{\gamma-1}\right)\frac{q_*}{kT_*} . \tag{13}$$

Physically, $m\lambda_*$ can be interpreted as the mass of the stopping medium at the sonic radius.

Since $T' = v' = 1$ at the sonic radius $r' = 1$, in order that dv'/dr' be definite, the term within the square bracket must vanish, i.e.

$$\frac{\Lambda_*\lambda_*}{2} = 1 \quad , \tag{14}$$

or

$$2\pi\left(\frac{\gamma-1}{\gamma}\right)\frac{q_*\rho^* r_*{}^3\Lambda_*}{kT_*G} - 1 = 0 \tag{15}$$

The physical meaning of Eq. (14) is clear if we write it in the alternative form:

$$4\pi r_*^2 q_* = 2\,\frac{G}{\rho_*\Lambda_* r_*}\cdot\frac{\gamma kT_*}{\gamma-1} . \tag{16}$$

Since $\rho_*\Lambda_* r_*$ is the total mass of the ablated material at the sonic radius, apart from the factor 2, the right-hand side simply expresses the thermal energy of the ablated material at the sonic radius. One recalls that the flow becomes sonic when an equipartition exists between its kinetic and thermal energies. Accordingly, Eq. (15) is nothing more than the requirement of energy conservation at the sonic radius.

Using Eq. (14) and omitting the prime, the system of equations (1)-(6) can be written in their alternative dimensionless form:

$$\frac{dv}{dr} = 2\,\frac{vT}{(T-v^2)}\left[\frac{\Lambda q}{Tv} - \frac{1}{r}\right] \tag{17}$$

$$\frac{dT}{dr} = 2\frac{\Lambda q}{v} - (\gamma-1)v\frac{dv}{dr} \quad , \tag{18}$$

$$\frac{dq}{dr} = \lambda_*\frac{\Lambda q}{r^2 v} \quad , \tag{19}$$

$$\frac{dE}{dr} = 2\lambda_*\left(\frac{L}{E_*\Lambda_*}\right)\frac{1}{r^2 v} \quad , \tag{20}$$

$$\rho v r^2 = 1 \quad , \tag{21}$$

$$p = \rho T \quad . \tag{22}$$

The Mach number, M of the flow is

$$M = (v^2/T)^{1/2} \tag{23}$$

It must be remembered that in evaluating the loss function, L, the original dimensional variable, E, should be used. The dimensionless cross section, Λ, is defined as

$$\Lambda = \Lambda(E)/\Lambda(E_*) \quad . \tag{24}$$

Once Λ(E) and L(E), or their equivalent σ(E) and L(E)/E, - namely the cross section of elastic scattering and of the inelastic slowing-down process - are given, the system of equations contains two parameters,

$$\Lambda_* \equiv \Lambda(E) \quad , \tag{25}$$

$$\lambda_* \equiv \rho_*\Lambda_* r_*/_m \quad , \tag{26}$$

i.e. the solution of the problem depends on the energy, E_*, of the incident electron and the mass, $m_{\lambda *}$, of the ablatant at the sonic radius.

In the original model of Parks and Turnbull, the following boundary conditions were used.

At the pellet surface ($r = \hat{r}$)

$$q = \hat{q} = 0 \qquad T = \hat{T} = 0 \tag{27}$$

At the far downstream of the flow ($r \to \infty$)

$$p \to 0$$

$$q \to \tilde{q} = q_O/q_*$$

$$E \to \tilde{E} = E_O/E_* \quad \text{where } E_O = 2kT_O \tag{28}$$

It should be recalled that according to the previous assumption (iv), q_O is related to the electron energy flux, q_∞, of the ambient plasma by $q_O = f_B f_H q_\infty$, where f_B and f_H are reduction factors due to the magnetic field and inelastic collision process other than that which increases the total enthalphy of the ablatant. ($f_B = 0.5$, $f_H = 0.6$ were taken in their original model).

2.2. Method of solution

Owing to the presence of a singularity at the sonic radius of Eq. (17) the system of equations has to be solved by a suitable adjustment of $(dv/dr)_{r=1}$ to meet the required boundary conditions (Eqs. (27) and (28)). Denoting $(dv/dr)_{r=1} = z_s$ and applying the L'Hôpital's rule to Eq. (17), it can be shown that[9]

$$z_s = \left(\frac{3-\gamma}{1+\gamma}\right) \left\{1 \pm \sqrt{1-2\frac{1+\gamma}{(3-\gamma)^2}\left[\lambda_*-1+\left(\frac{d\Lambda}{dr}\right)_{r=1}\right]}\right\} \quad . \tag{29}$$

$(d\Lambda/dr)_{r=1}$ is a function of E_*[9], the incident electron energy at the sonic radius; once E_* is given, z_s then depends on the choice of λ_*.

To facilitate the guesswork, we shall replace λ_* by K[13], and so

$$K = \lambda_* + \left(\frac{d\Lambda}{dr}\right)_{r=1} \quad . \tag{30}$$

It can be shown that for z_s to be definite and positive, K must be chosen in the open interval $(0,1)$[13].

For a given E_*, after a value of K is chosen, all the derivatives of the variables, v, T, E, and q, at the sonic radius will be known. The system of equations, (17)-(20), are first integrated inwardly, until the boundary condition (Eq. (27)) is satisfied, thus locating the pellet surface

$$\hat{r} = r_p/r_* \tag{31}$$

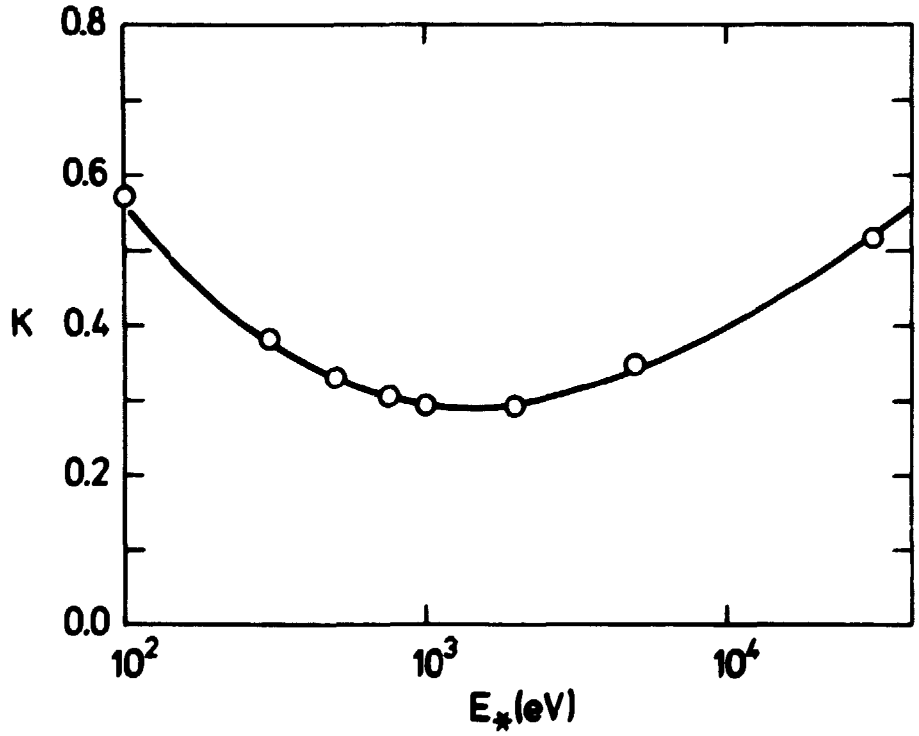


Fig. 1. The eigenvalue, K, as a function of the incident electron energy, E_*, at the sonic radius corresponding to the boundary condition at the pellet surface, $\hat{q} = 0$ and $\hat{T} = 0$.

The system of equations is then integrated outwardly to satisfy the boundary condition (Eq. (28)) far downstream. If the value of $\tilde{E}$, or the corresponding value $E_o/2 = \tilde{E} \cdot E_*/2$, does not match the value of the ambient plasma temperature, kT_o, a new value of K has to be tried. Values of K, $\tilde{E}$, $\tilde{q}$, and $\hat{r}$ obtained in this way for a given E_* are shown in Figs. 1 and 2. As an illustrative example, we have taken $E_* = 3 \times 10^4$ eV and plotted in Fig. 3 the variation of the energy, E/E_*, the flux, q/q_*, and the Mach number, M, of the ablated flow with respect to the radial distance of expansion, r/r_*.

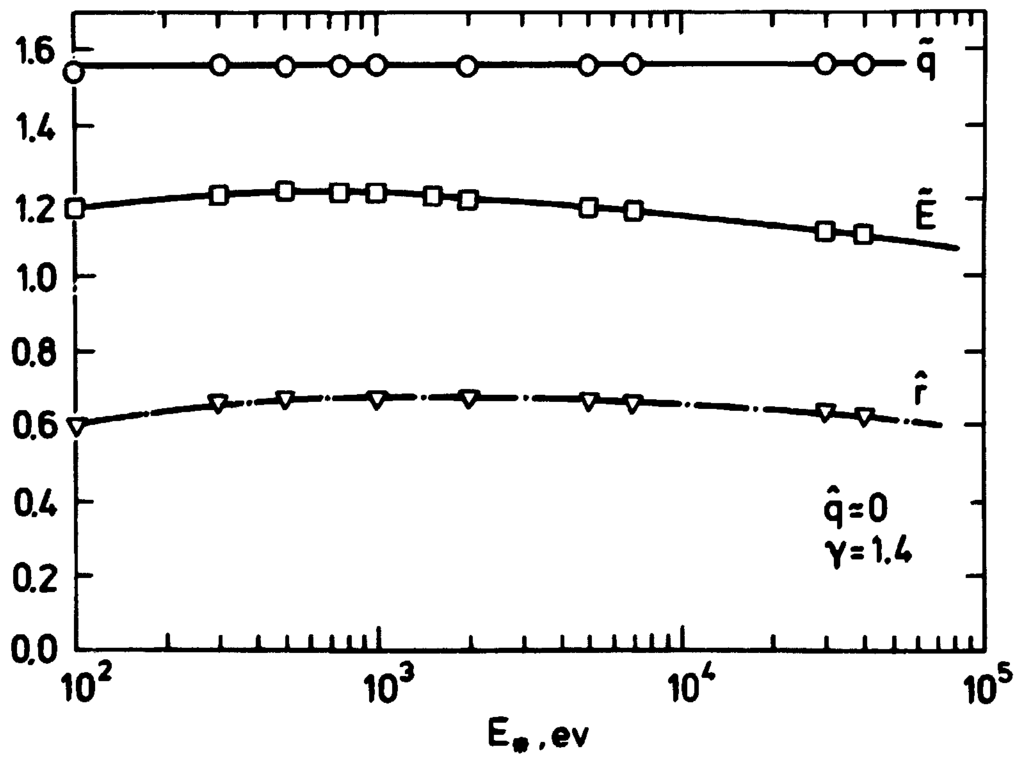


Fig. 2. Variation of the normalized initial electron energy $\tilde{E} = E_o/E_*$ and energy flux $\tilde{q} = q_o/q_*$, at points far downstream, and variation of the normalized pellet radius, $\hat{r} = r_p/r_*$, with respect to the incident electron energy, E_*, at the sonic radius, r_*, corresponding to the boundary condition, $\hat{q} = 0$ and $\hat{T} = 0$.

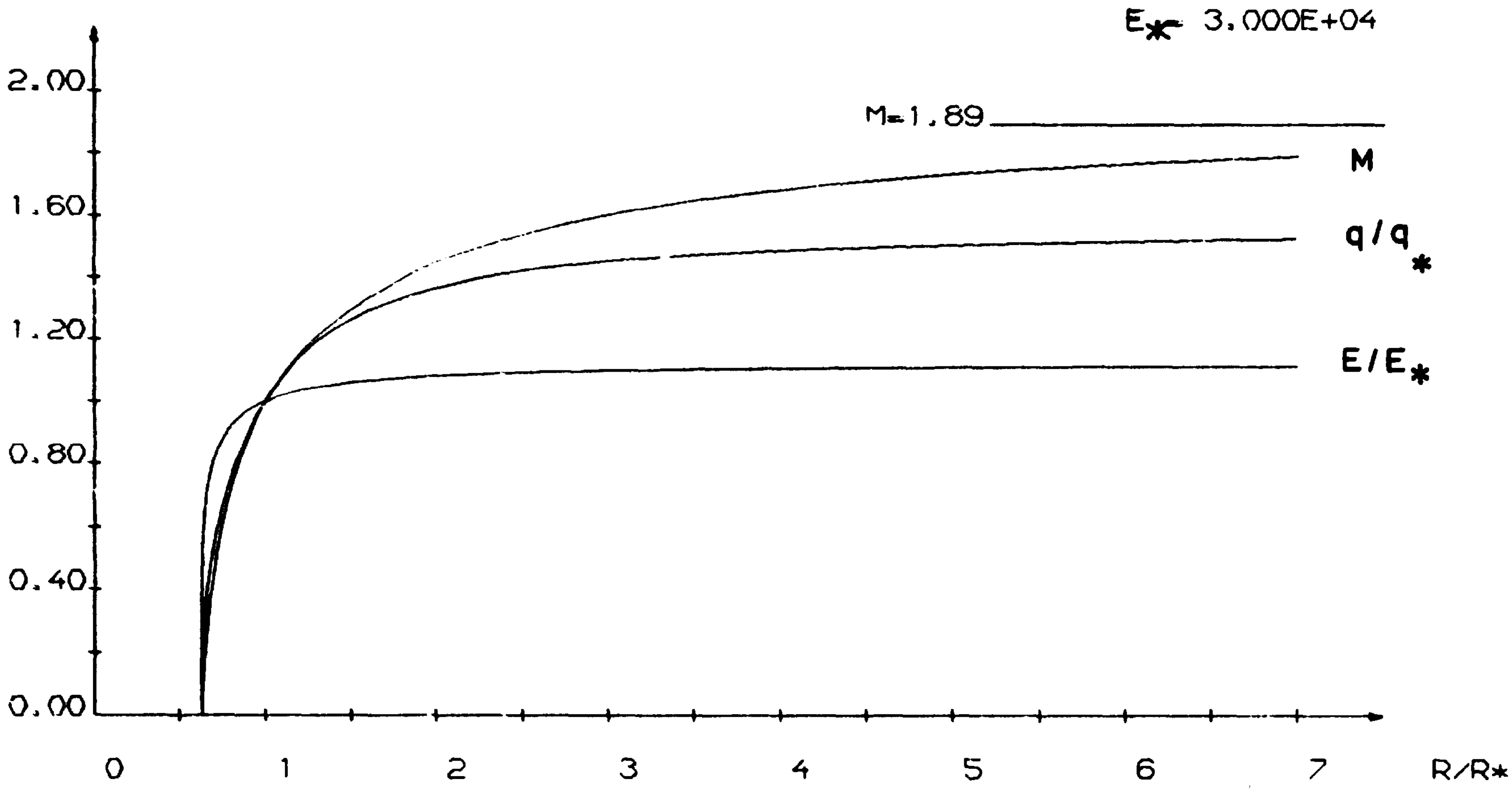


Fig. 3. Normalized incident electron energy, E/E_*, and energy flux, q/q_*, versus normalized radial distance, R/R_*. $E_* = 3\times10^4$ eV is the incident electron energy at the sonic radius, $R/R_* = 0.629$ is the pellet location.

2.3. Discussion of the results

As a consequence of the boundary condition (Eq. (27)) imposed at the pellet surface, the particle density, n_v, of the evaporate at the pellet surface always exceeds the density, n_s, ($\approx 3\times10^{22}$ cm^{-3}) of the solid hydrogen for practical ranges of interest of the plasma temperature, kT_o. For a given E_*, the eigenvalue is K (or its equivalent, λ_*). Hence, the solution of the system of equations (Eqs. (17)-(23)) depends only on the plasma temperature, $kT_o = E_*\bar{E}/2$, and is universally valid for arbitrary values of the plasma density, n_o, and pellet radius, r_p.

From the law of conservation of mass, once the sonic radius, r_*, is known, the pellet ablation rate, G, is determined by the state of the ablatant at the sonic radius; thus

$$G = 4\pi r_*^2 \ \rho_* \left(\frac{\gamma k T_*}{m}\right)^{1/2} \tag{32}$$

The dependence of the abaltaion rate, G, on the pellet radius, r_p, and the ambient plasma state, T_o, and n_o appears indirectly through the dependence of r_*, ρ_* and T_* on these quantities, thus

$$r_* = r_p/\hat{r} \quad , \qquad \hat{r} = \hat{r}(E_*) \quad , \tag{33}$$

$$\rho_* = \frac{m\lambda_*}{r_*\Lambda_*} \quad , \qquad \lambda_* = \lambda_*(E_*), \ \Lambda_* = \Lambda_*(E_*) \quad , \tag{34}$$

$$kT_* = \frac{m^{1/3}}{\gamma}\left[\frac{(\gamma-1)r_*\Lambda_* q_*}{2}\right]^{2/3} \quad , \text{ and} \tag{35}$$

$$q_* = q_0/\tilde{q} = \frac{n_0(2kT_0)^{3/2}}{(4\pi m_e)^{1/2}} \Big/ \tilde{q}, \qquad \tilde{q} = \tilde{q}(E_*) \quad . \tag{36}$$

Using Eqs. (12) and (15), a scaling law of the particle ablation rate, $\dot{N}_a$, can be written as

$$\begin{aligned} \dot{N}_a &= G/m \\ &= 1.854 \times 10^7 \frac{\lambda_*}{\Lambda_*^{2/3}} \frac{n_0^{1/3}(kT)^{1/2}}{(\tilde{q})^{1/3}} \left(\frac{r_p}{\hat{r}}\right)^{4/3} \quad . \end{aligned} \tag{37}$$

The dependence of N_a on the plasma temperature, kT_0, however, is given implicitly through the dependence of λ_*, Λ_*, $\tilde{q}$, and $\hat{r}$ on E_*.

3. FORMULATION OF ALTERNATIVE BOUNDARY CONDITIONS

3.1. Boundary condition at far downstream point

While the boundary condition Eq. (28) might be satisfactory for analytical solutions, it poses the difficulty of stopping the integration process for numerical computations. This is because E and q seldom approach their constant values of $\tilde{E}$ and $\tilde{q}$ at the same rate.

This difficulty can be resolved easily by noting that at a sufficiently large value of r, asymptotic solutions of the flow parameters exist. Specifically, we have[9,13]

$$\tilde{V} = \left(\frac{30\tilde{\Lambda}\tilde{q}}{7\gamma-5}\right)^{1/3} r^{1/3} \quad , \tag{38}$$

$$\tilde{T} = \frac{\gamma}{5}\left(\frac{30\tilde{\Lambda}\tilde{q}}{7\gamma-5}\right)^{1/3} r^{2/3} \quad . \tag{39}$$

Consequently, as an alternative boundary condition for the flow far downstream, we may replace it by the requirement that

$$M \simeq \sqrt{5/\gamma} \quad \text{for} \quad r > \tilde{r} \tag{40}$$

3.2. Implication of the boundary condition $\hat{q} = 0$ and $\hat{T} = 0$

As the integration of the system of equations, (17)-(20), continuously progresses from the sonic radius r=1 towards the pellet surface, $r = \hat{r}$, it is difficult to conceive q and T becoming vanishingly small at the same rate. This poses a problem regarding the choice of the eigenvalue K. To resolve this uncertainty, it is necessary to examine in detail the vaporization process occurring at the pellet surface.

Assuming that most of the energy delivered to the pellet is in the vaporized phase, the law of energy conservation can be written as

$$4\pi r_p^2 \, q_p = G \, H_a \tag{41}$$

where

$H_a = h_e + \frac{v_e^2}{2} + \varepsilon$	is the total heat of the ablation process,
$h_e = \frac{\gamma}{\gamma-1} \frac{kT_v}{m}$	is the enthalpy of the evaporate,
ε	is the sublimation energy per gm of hydrogen molecules.

Solving for G, we obtain

$$G = 4\pi r_p^2 \frac{q_p}{H_a}. \tag{42}$$

Since $\hat{q} = q_p/q_*$ by definition, for any finite ablation rate, G, the boundary condition $\hat{q} \rightarrow 0$ then infers that $H_a \rightarrow 0$ also. The minimum energy required for the ablation to occur is the sublimation energy, ε; thus, the boundary condition $\hat{q} \rightarrow 0$, although mathematically reasonable, is physically unrealistic.

Substituting G given by Eq. (15) into (42), and solving for q_p, we obtain

$$q_p = \frac{H_a}{2r_p^2}\left(\frac{\gamma-1}{\gamma}\right)\rho_*\Lambda_* r_*^3 \frac{q_*}{kT_*} \quad . \tag{43}$$

Using the definitions of $\hat{q}(= q_p/q_*)$, $\hat{T}(=T_v/T_*)$, $\hat{r}$, λ_* and h_e, it is possible to derive the following relationship from Eq. (43):

$$\frac{\hat{T}}{\hat{q}} = 2\frac{\hat{r}^2}{\lambda_*}\frac{h_e}{H_a} \tag{44}$$

Extensive computational work indicated for a practical range of interest of the plasma temperature: $\hat{r} < 0.7$, $\lambda_* > 0.9$. Thus,

$$\frac{\hat{T}}{\hat{q}} \lesssim \frac{h_e}{H_a} \tag{45}$$

Since $H_a > h_e$ always, Eq. (45) requires that the proper value of K must be chosen such that $\hat{T} < \hat{q}$. In other words, K must be chosen from the left branch of the cusps of the curves of $\hat{T}$ and $\hat{q}$ vs. K. Thus, as shown in Fig. 4, $K \lesssim 0.290$ is required for $E_* = 1\times10^3$ eV.

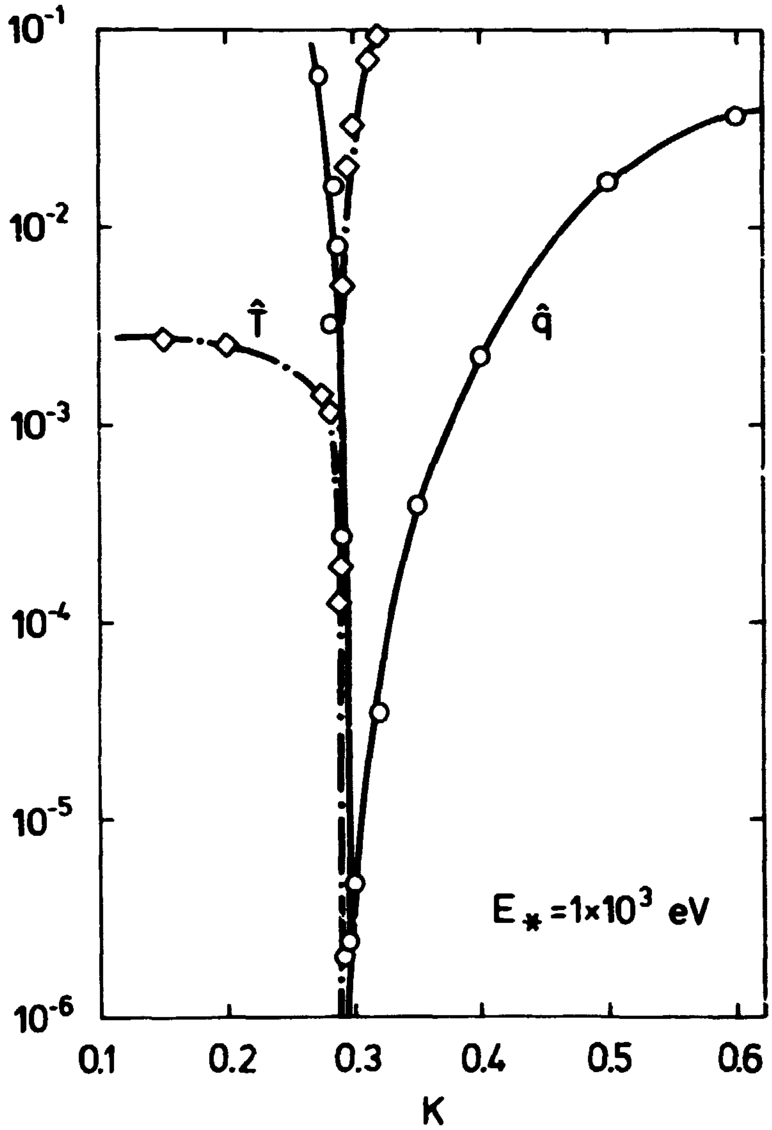


Fig. 4. The normalized temperature, $\hat{T} = T_v/T_*$, of the ablatant, and the normalized incident electron energy flux, $\hat{q} = q_p/q_*$, at the pellet surface as functions of the eigenvalue, K.

3.3. Mode of the vaporization process

3.3.1. Ablation by slow sublimation

According to kinetic theory, sublimation occurs when the kinetic energy of a molecule moving normally to the surface exceeds the surface potential, U_o, of the solid. Within a good degree of approximation, one may take the sublimation energy of the solid, ε' (= ε/m), as the surface potential. For molecules with a velocity distribution characterized by the surface temperature, T_s, the evaporated particle flux, ϕ, according to Frenkel[14] is given by

$$\phi = n'\nu_o e^{-U_o/kT_s} \quad , \tag{46}$$

where n' is the number of molecules per unit surface area, ν_o = kT_s/h, the frequency of free vibration of the surface molecules in the bound state about their equilibrium positions, and k and h the Boltzmann and Planck constants, respectively. Substituting $U_o = \varepsilon' = 0.01$ eV, $n' = 3.02\times10^{14}$ cm^{-2} for solid hydrogen, and taking $T_s = 10^o$ K in Eq. (46), we obtain

$$\phi = 3.811\times10^{23} \text{ cm}^{-2} \text{ sec}^{-1} \quad . \tag{47}$$

The time required to ablate a pellet of radius, r_p, is then given by

$$t_s = \int_o^{r_p} \frac{4\pi r^2 n_s dr}{4\pi r^2 \phi} = 7.872\times10^{-2}\ r_p \quad . \tag{48}$$

In the above, we have taken the particle density of solid hydrogen, $n_s = 3\times10^{22}$ cm^{-3}.

Using the neutral-shielding model, we have considered the injection of a solid hydrogen pellet into a homogeneous plasma. The computed particle ablation rate, $\dot{N}_a$, corresponding to three representative tokamak discharges is listed in Table 1, where cases C and B correspond to plasma conditions of present-day small- and intermediate-size tokamaks. Case A, on the other hand, represents that of a future fusion reactor. For comparison, the ablation time, t_a, is tabulated along with the sublimation time, t_s, according to Eq. (48).

Table 1. Comparison of pellet ablation time, t_a, with sublimation time, t_s, in three representative tokamak discharges

	Case A	Case B	Case C
T_o, eV	1.68×10^4	6.07×10^2	3.04×10^2
n_{eo}, cm^{-3}	1×10^{14}	2×10^{13}	2×10^{13}
r_p, cm	0.4	0.02	0.03
$\dot{N}_a$, sec^{-1}	6.403×10^{26}	2.634×10^{22}	1.583×10^{22}
t_a, sec	1.256×10^{-5}	3.817×10^{-5}	2.143×10^{-4}
t_s, sec	3.149×10^{-2}	1.574×10^{-3}	2.362×10^{-3}
T_v,deg. K	0.539	0.557	0.128

One observes that the temperature, T_v, of the ablated vapour is below 1^o K in all three cases considered. It is therefore unlikely that T_s will be much higher than 10^o K. This indicates that sublimation takes place far too slowly to account for the ablation of the pellet under the present circumstances.

3.3.2. Dynamic phase transition and alternative boundary condition at the pellet surface

In the previous section, we have shown that the ablation of a solid hydrogen pellet in tokamak discharges cannot be accomplished through the sublimation process. The reason for this is that in order to continue the sublimation process after a thin surface layer has evaporated, the sublayer must be raised to the surface temperature through the diffusion of the heat source. Instead of proceeding by slow diffusion, the transport of heat can be accomplished more effectively through the propagation of an evaporation front. In the present situation, therefore, the evaporation process is a dynamic phase transition.

We denote the condensed and the vaporization phases of the pellet by the subscript c and r, respectively. In the frame of references relative to the moving evaporating front, shown in

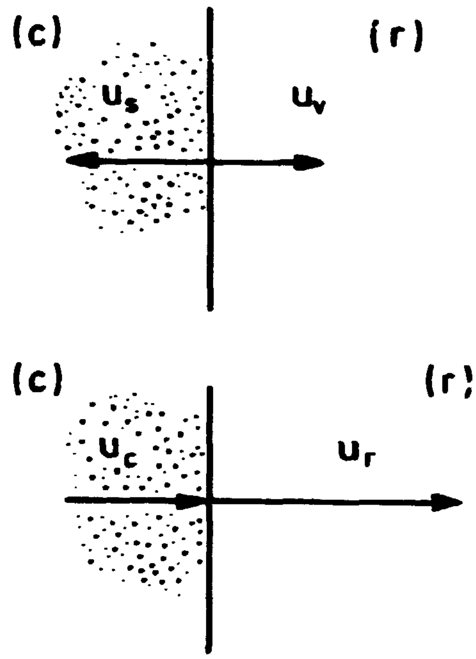


Fig. 5. Schematic drawing of the evaporation process at the pellet surface. u_s and u_v are the respective velocities of the evaporating front and ablated vapor in the laboratory frame of reference. u_c and u_r are the respective velocities of the condensed matter and evaporate relative to the moving evaporating front.

Fig. 5, the three conservation laws of mass, momentum, and energy can br written as

$$\rho_c u_c = \rho_r u_r = \dot{m} \quad , \tag{49}$$

$$p_1 + \rho_c u_c^2 = p_r + \rho_r u_r^2 \quad , \tag{50}$$

$$e_c + \frac{p_1}{\rho_c} + \frac{u_c^2}{2} + \frac{\psi^2}{2} = e_r + \frac{p_r}{\rho_r} + \frac{u_r^2}{2} \quad , \tag{51}$$

where

$$p_1 = p_c - p_o \quad , \tag{52}$$

$$\frac{\psi^2}{2} = \frac{q_p}{\dot{M}} - \varepsilon \quad , \tag{53}$$

$$u_c = - u_s \quad , \tag{54}$$

$$u_r = u_c + u_v \quad . \tag{55}$$

In the above system of equations, u_s is the propagating speed of the ablating front in the laboratory frame of reference, u_r the velocity of the evaporate with respect to the moving ablation front, p_o the ambient pressure at a point far downsteam, q_p the energy flux at the pellet surface, ε the sublimation energy per gram of hydrogen molecules, and e_c and e_r the internal energies of the condensed and vaporized phases, respectively. The remaining notations are self-explanatory.

Assuming the evaporate to be a perfect gas, we have

$$e_r = \frac{1}{\gamma-1} p_r/\rho_r \quad , \tag{56}$$

where γ is the ratio of the constant pressure and constant volume specific heats Eqs. (49)-(51) in combination with the Jouget condition (Eq. (57)).

$$u_r^2 = \frac{\gamma p_r}{\rho_r} = \frac{\gamma k T_v}{m} \tag{57}$$

can be reduced to a single quadratic equation of the compression ratio ρ_c/ρ_r, or u_r/u_c[15]; thus,

$$\left(\frac{\gamma+1}{\gamma-1}\right)\left(\frac{u_r}{u_c}\right)^2 - 2\left(\frac{\gamma+1}{\gamma}\right)\left(\frac{u_r}{u_c}\right) - \frac{\frac{\psi^2}{2}+e_c}{\frac{u_c^2}{2}} - 1 = 0 \quad . \tag{58}$$

under the assumption of $u_c^2/2 \gg e_c$ (which can be considered as the condition for the occurrence of a dynamic phase transition), this equation, in terms of the rarefaction ratio,

$$\xi = u_c/u_r = \rho_r/\rho_c \quad , \tag{59}$$

becomes

$$\xi^2 - 2\left(\frac{\gamma+1}{\gamma}\right)\xi + \left[\frac{\gamma+1}{\gamma-1} - \left(\frac{\psi}{u_r}\right)^2\right] = 0 \quad . \tag{60}$$

Solving for ξ, we obtain

$$\xi = \left(\frac{\gamma+1}{\gamma}\right)\left\{1 \pm \left[\left(\frac{\psi}{\frac{\gamma+1}{\gamma}u_r}\right)^2 - \frac{1}{\gamma^2-1}\right]^{1/2}\right\} \quad . \tag{61}$$

Since we have already fixed the sign of u_c, the physically meaningful solution of ξ will be limited to

$$0 < \xi < 1 \quad , \tag{62}$$

Where $\xi > 0$ (i.e. $u_c > 0$, $u_r > 0$) infers that the ablation front is moving inwardly from the pellet surface ($u_s = -u_c$). From Eq. (62), the condition of $\xi < 1$ (or $u_c < u_r$), and the restriction of $u_r > 0$, it follows that the ablated flow is streaming away from the pellet surface ($u_v > 0$). We would like to remark here that although $\xi < 1$ also indicates that $\rho_r < \rho_c$, this condition does not necessarily also require that $\rho_r < \rho_s$, the original uncompressed density of solid hydrogen.

Following these considerations, we shall restrict our discussion to the negative branch of ξ in Eq. (61) only; the condition of Eq. (62) then further requires that

$$\frac{2}{\gamma(\gamma-1)} < \left(\frac{\psi}{u_r}\right)^2 < \frac{\gamma+1}{\gamma-1} \quad . \tag{63}$$

Using the definition of ψ^2 (Eq. (53)) and the Jouget condition (Eq. (57)) we can write $(\psi/u_r)^2$ as

$$\left(\frac{\psi}{u_r}\right)^2 = \frac{2}{\gamma}\,\frac{q_p-q_s}{\dot{n}kT_v} \quad , \tag{64}$$

where

$$\dot{n} = \dot{M}/m \quad , \tag{65}$$

$$q_s = \dot{n}\,\varepsilon' \quad , \tag{66}$$

Thus, $\dot{n}$ is the ablated particle flux; q_s is the energy flux of the evaporate corresponding to the vapourization process itself, whereas $\dot{n}kT_v$ is the thermal energy flux of the ablated vapour.

Substituting Eq. (64) into (63) we obtain an alternative boundary condition at the pellet surface; thus,

$$\frac{\gamma}{2}\left(\frac{\gamma+1}{\gamma-1}\right) > \frac{q_p-q_s}{\dot{n}\,kT_v} > \frac{1}{\gamma-1} \quad . \tag{67}$$

We define a number,

$$CR = \frac{q_p-q_s}{\dot{n}\,kT_v} \quad . \tag{68}$$

Thus, for dynamic phase transition to occur, we must have

$$4.2 > CR > 2.5 \tag{69}$$

for an evaporate of diatomic molecules.

4. COMPUTATIONAL RESULTS USING THE ALTERNATIVE BOUNDARY CONDITIONS

Computational results given in Table 2 indicate that when the boundary condition (Eq. (69)) is used, the eigenvalue of K depends not only on E_*, the attenuated energy of the incident electron at the sonic radius, but also on the ambient plasma density, n_o, and the pellet radius, r_p.

Table 2. Dependence of the eigenvalue, K, on plasma density, n_o, and pellet radius, r_p, at $E_* = 3\times10^4$ eV

	$n_o = 2\times10^{13}$ cm^{-3} $r_p = 2$ mm	$n_o = 2\times10^{13}$ cm^{-3} $r_p = 4$ mm	$n_o = 1\times10^{14}$ cm^{-3} $r_p = 4$ mm
K	0.51640	0.51690	0.517435
CR	3.926	2.928	3.131
$\hat{n}=q_p/q_o$	2.1459×10^{-3}	1.3527×10^{-3}	4.7242×10^{-4}
T_o, eV	1.6776×10^4	1.6778×10^4	1.6780×10^4
$\dot{N}_a$, sec^1	1.4815×10^{26}	3.7385×10^{26}	6.4026×10^{26}

From the results shown in Table 2, it is easy to verify that in spite of the replacement of the boundary condition (Eq. (27)) by the alternative condition (Eq. (69)), the scaling law of the pellet ablation rate, $\dot{N}_a$, still holds.

In order to compare the results obtained by using the two different boundary conditions, Eqs. (27) and (69), we have chosen three different set values of E_*, n_o, and r_p corresponding to pellet injection within the possible range of tokamak discharges. The results are summarized in Table 3.

Table 3. Comparison of the ablatant state corresponding to the two different boundary conditions of $\hat{q} = 0$ and $\hat{q} \neq 0$ (or $2.5 < CR < 4.2$) at the pellet surface.

E_*, eV	3^{4}		2^{3}		5^{2}	
n_o, cm^{-3}	1^{14}		2^{13}		2^{13}	
r_p, cm	0.4		0.2		0.03	
	$\hat{q}$=0	CR=3.131	$\hat{q}$=0	CR=3.168	$\hat{q}$=0	CR=3.986
T_o,eV	1.6780^{4}	1.6780^{4}	1.2036^{3}	1.2033^{3}	3.0426^{2}	3.0400^{2}
r_*/r_p	1.5903	1.5901	1.4815	1.4810	1.5078	1.5058
H_a,eV	6.6860^{-4}	1.0790^{-2}	1.4280^{-3}	1.0344^{-2}	5.8813^{-4}	1.0826^{-2}
T_v,eV	4.6421^{-5}	2.5277^{-4}	7.3527^{-5}	1.0846^{-4}	1.1004^{-5}	2.0727^{-4}
n_v,cm^{-3}	2.5706^{25}	4.7276^{24}	1.0941^{23}	7.4071^{22}	1.6563^{23}	8.7076^{21}
T_*eV	4.9487	4.9484	1.6489	1.6489	5.1747^{-1}	5.1723^{-1}
n_*,cm^{-3}	6.9161^{19}	6.9141^{19}	1.4733^{18}	1.4714^{18}	1.0522^{18}	1.0482^{18}
T_F,eV	8.6586^{1}	8.6574^{1}	1.6268^{1}	1.6269^{1}	5.5064	5.5047
n_F,cm^{-3}	2.3132^{15}	2.3126^{15}	2.9056^{14}	2.9001^{14}	1.6348^{14}	1.6255^{14}
$\tilde{r}(=r_F/r_*)$	6.1528^{1}	6.1528^{1}	2.9239^{1}	2.9247^{1}	3.2323^{1}	3.2352^{1}
η	2.9294^{-5}	4.7242^{-4}	1.7377^{-4}	1.2569^{-3}	2.3337^{-4}	4.2727^{-1}
$\dot{N}_a$,sec^{-1}	6.4075^{26}	6.4026^{26}	1.7092^{24}	1.7062^{24}	1.5939^{22}	1.5833^{22}
K	0.5177	0.517435	0.2912	0.29076	0.3310	0.3293

Note: The index appearing above is an abbreviation of the power of 10, thus 2^{13} means 2×10^{13}.

In Table 3, $\hat{r}(= r_p/r_*)$ is the normalized pellet radius and $\tilde{r}(= r_F/r_*)$ the normalized radial distance at a point far downstream where the Mach number, M, of the flow is within 0.1 percent of its asymptotic value as given by Eq. (40). The energy attenuation factor is denoted by $\eta(= q_p/q_o)$ and the particle ablation rate by $\dot{N}_a$. To distinguish the temperature and density of the ablatant at various radial locations, subscript notations are employed; thus, "v" denotes their values at the pellet radius, and "F" at the far downstream location $\tilde{r}$.

From the tabulated results, some general trends between the solutions of the two cases can be observed. On comparing the results of $\hat{q} \neq 0$ (denoted by their corresponding values of the number CR) with the case of $\hat{q} = 0$ we find that the energy at-attenuation factor, η, and the temperature, T_v, of the vapour at the pellet surface is about an order of magnitude higher in the case of the inequality, while the particle density is about an order of magnitude lower. The heat of ablation, H_a, for case $\hat{q} = 0$ case is well below the sublimation energy of solid hydrogen, $\varepsilon' = 0.01$ eV. In the $\hat{q} \neq 0$ case, on the otherhand H_a is only slightly above ε', i.e. only 3 to 8% of the energy corresponding to the sublimation is used up in expanding and heating the evaporature. As further heating and expanding of the evaporate is due to the degraded incident electrons, by the time the evaporate reaches the sonic surface and beyond, there is practically no difference in the state of the ablatant, corresponding to the two cases.

5. SUMMARY AND DISCUSSION

Anticipating the existence of an effective shielding mechanism, Parks and Trunbull reasoned that q_p, the energy flux received, and T_v, the temperature of the evaporate at the pellet surface, must be considerably less than their corresponding values of q_* and T_* at the sonic radius of expansion. Consequently, to analyze the flow field of the ablatant, they used

$$\hat{q} = q_p/q_* = 0 \quad \text{and} \quad T = \hat{T}_v/T_* = 0$$

as the appropriate boundary condition at the pellet surface. This boundary condition, which assumes implicity that the evaporation of the pellet is a sublimation process, is physically unsatisfactory in two aspects. The choice of the eigenvalue, K, of the system of governing equations, and hence the solution of the problem, depends on the ambient plasma temperature only and is universally valid for arbitrary values of plasma density and pellet radius. The condition $\hat{q} = 0$ does not conform to the requirement of energy conservation.

By comparing the time required to sublimate a pellet of radius r_p with the computed ablation time according to the neutral shielding model, it was shown that the ablation of a hydrogen pellet in present and future tokamak discharges is a dynamic phase transition - i.e. the transport of heat is due to the propagation front instead of to a slow sublimation process. Accordingly, an alternative boundary condition, Eq. (69), consistent with the requirement of energy conservation, is formulated. The corresponding eigenvalue, K, is seen to depend not only on the plasma temperature, but also on the plasma density and pellet radius. Computational results based on the new boundary condition showed that compared with results obtained from the previous condition of vanishing energy flux, $\hat{q} = 0$, the ablatant in the subsonic region is hotter and less dense and the energy flux, q_p, received at the pellet surface is higher. The particle density, n_v, of the vapour at the pellet surface, except in the case of relatively cold discharges ($T \lesssim 300$ eV) is still higher than the orginal density of solid hydrogen, n_s. This unusually dense state of the ablatant may well be attributed to the inadequacy of assuming the ablated vapour to be an ideal gas. Nevertheless, it should be noted that the result does not violate the requirement of $\rho_r < \rho_c$. The reason for this is that in the present situation the pellet material is actually being compressed by the evaporating front. (It may be remarked here that due to its absorption of heat at the pellet surface, even the case of $\rho_r > \rho_c$ does not violate the second law of thermodynamics. Our requirements of $\rho_r < \rho_c$ is based on the condition that the ablated flow must stream away from the pellet surface).

With respect to the two cases of the boundary conditions used, there is a most significant difference between the state of the ablatant near the pellet surface. The discrepancy becomes less and less noticeable once the ablatant passes beyond the sonic radius. Since the pellet ablation rate, according to Eq. (15), and less noticeable once the ablatant passes beyond the sonic radius. Since the pellet ablation rate, according to Eq. (15), depends on the state of the ablatant and the energy deposition at the sonic radius, the scaling law of the ablation rate is not affected by this change of the boundary condition. The insensitivity of the solution of the problem to the boundary condition at the pellet surface can be explained most readily by following the course of the incoming electrons. As the stream of incoming electrons approaches the pellet, the associated energy flux is only slightly attenuated in the supersonic region. A strong reduction of the energy flux of the incoming electrons occurs only when the stream enters the subsonic region. Due to the steep drop of the energy flux in the vicinity of the pellet surface, the value of the energy flux, q_p, at the pellet surface is quite arbitrary for a definite pellet ablation rate. The arbitrariness in the choice of the proper value of q_p or $\hat{q}$ can be illustrated in the following alternative way of selecting the eigenvalue, K. At a given value of E_*, K is now chosen to satisfy the two conditions:

$$\hat{T} < \hat{q}, \text{ and}$$

$$\hat{q} < 5\times10^{-2} \quad .$$

The first condition, as shown in Section 3.2, is a consequence of energy conservation, and the second follows from the expectation of the existence of an effective shielding mechanism.

Using the above conditions, computations were carried out for a 0.2-mm radius hydrogen pellet injected into a homogeneous plasma of particle density $n_o = 2\times10^3$ cm^{-3}. The results are shown in Figs. 6 and 7. For a given $E_* = 1\times10^3$ eV, the permissible value of K is to be found within the interval [0.276 - 0,290], marked by the dashed lines. From Fig. 6., one observes that for allowable values of K, the energy flux, q_p, can vary by more than four orders of magnitude. On the other hand T_*, the temperature of the ablatant at the sonic radius, as shown in Fig. 7, is almost constant; the variation of the plasma temperature, T_o, and the pellet ablation rate, $\dot{N}_a$, in the same interval of K, is scarcely of any practical importance.

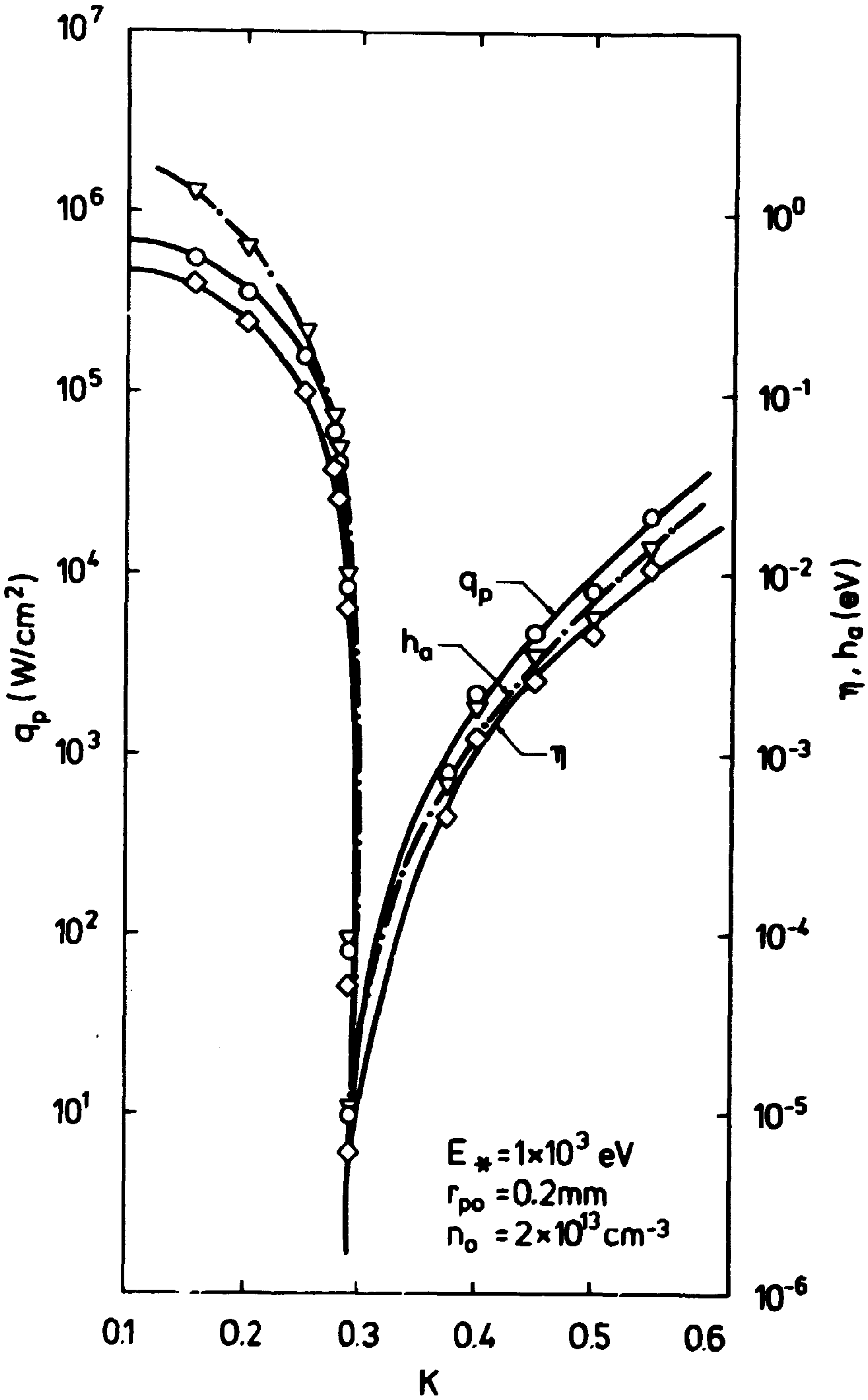
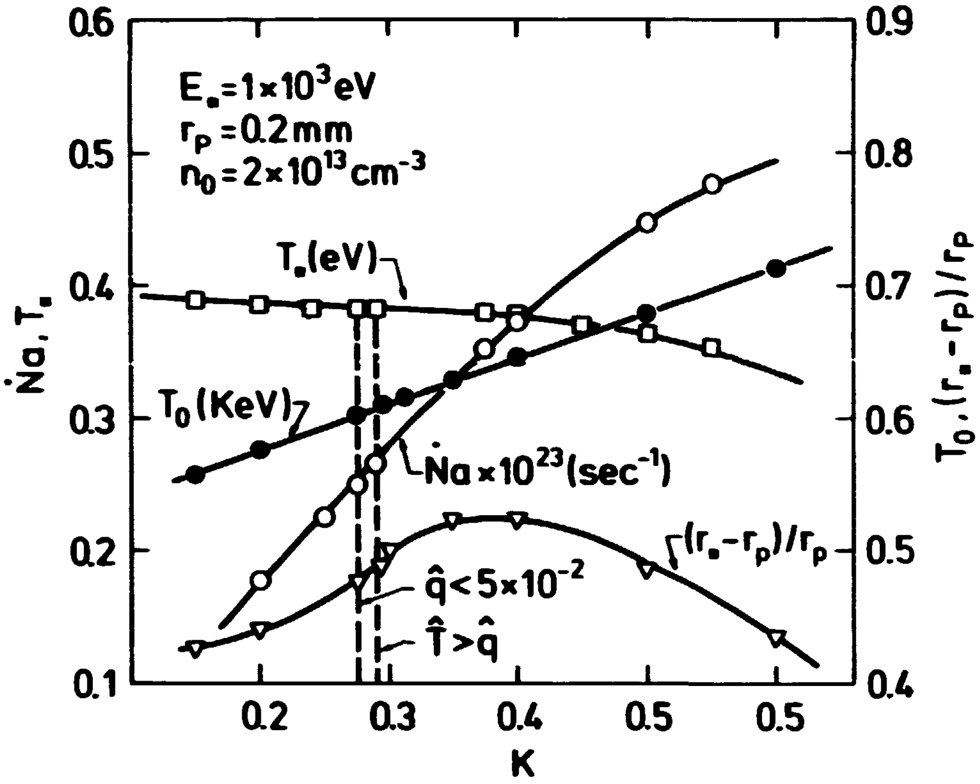


Fig. 6. Variation of the total heat of ablation, h_a, incident electron energy flux, q_p, at the pellet surface, and the energy attenuation factor, $\eta = q_p/q_o$, with respect to the eigenvalue, K, at the given condition, $E_* = 1\times10^3$ eV, $n_o = 2\times10^{13}$ cm^{-3}, and $r_p = 0.2$ mm. The corresponding values of h_a, q_p, and η satisfying the boundary condition, $\hat{T} < \hat{q}$ and $\hat{q} < 5\times10^{-2}$, are to be found within the region marked by the two dashed lines.



<u>Fig. 7</u>. Variation of the pellet ablation rate, $\dot{N}_a$, and energy absorption layer, (r_*-r_p), with respect to the eigenvalue, K for the given condition, $E_* = 1\times10^3$ eV, $n_o = 2\times10^{13}$ cm^{-3}, and $r_p = 0.2$ mm. T_o and T_* are the incident electron temperature at points far downstream and at the sonic radius, respectively. Values of $\dot{N}_a$, (r_*-r_p), T_o, and T_* satisfying the boundary condition, $\hat{T} < \hat{q}$ and $\hat{q} < 5\times10^{-2}$, are to be found in the region marked by the two dashed lines.

In summary, our present analysis shows that within the framework of the assumptions made, the validity of the neutral shielding model is based mainly on the existence of a layer of dense ablated material of thickness less than one pellet radius surrounding the pellet in a region where strong energy absorption occurs. As long as this strong energy absorption mechnism exists, the actual vaporization process occurring at the pellet surface is of no practical relevance to the pellet ablation rate.

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